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The reciprocant represented by (6) may be written

$$1+\left(\frac{dy}{dx}\right)^2-3\left(\frac{d^2y}{dx^2}\right)^2\frac{dy}{dx}\Big/\frac{d^3y}{dx^3}=0\ldots.(\alpha).$$

After differentiating (α), etc., we have the reciprocant represented by (5).

From the equation $x^2+xy+y^2=1$ may be deduced by differentiation, etc.,

$$x,=-\left(\frac{2dy+dx}{dy+2dx}\right)y,=-\left(\frac{2(dy/dx)+1}{dy/dx+2}\right)y\ldots.(a);$$

$$\left(\frac{dy}{dx}+2\right)^2=-\left(2\frac{dy}{dx}+1\right)\left(\frac{dy}{dx}+2\right)\frac{dy}{dx}-3y\frac{d^2y}{dx^2}\ldots.(b);$$

$$3y=-\left[2\left(\frac{dy}{dc}\right)^3+6\left(\frac{dy}{dc}\right)^2+6\left(\frac{dy}{dc}\right)+4\right]\Big/\frac{d^2y}{dx^2}\ldots.(c).$$

$$\therefore 6\left(\frac{dy}{dx}\right)^2\left(\frac{d^2y}{dx^2}\right)^2+15\left(\frac{d^2y}{dx^2}\right)^2\frac{dy}{dx}+6\left(\frac{d^2y}{dx^2}\right)^2-2\left(\frac{dy}{dx}\right)^3\frac{d^3y}{dx^3}-6\left(\frac{dy}{dx}\right)^2\frac{d^3y}{dx^3}$$

$$-6\frac{dy}{dx}\cdot\frac{d^3y}{dx^3}-4\frac{d^3y}{dx^3},=T^3-\left(\frac{2A^2-3B}{B}\right)T^2-\left(\frac{5A^2-3B}{B}\right)T-2\left(\frac{A^2-B}{B}\right),=0$$

....(7), which is a *cubic* reciprocant. From (6) is deduced $T^2-2(A^2/B)T+1=0$, which is a *quadratic* reciprocant. Transforming (5), we have $T=A^2B/2(AC-B^2);=A^2/B,=5AB/2C,$ = etc., which are *linear* reciprocants. In so far as their number is concerned, the pure reciprocants are like the major planets of the solar system—few; while the mixed reciprocants are like the minor planets of the same system—many.

NOTE.—Since we had the good fortune to be one of Professor Sylvester's students—one of the last and probably the youngest he had—in the Johns Hopkins University, we declare that Dr. Halsted's biographical sketch of that *enthusiastic* mathematical professor and investigator is the acme of appropriateness; and we also declare that all the commendation which that sketch accords to Professor Sylvester, is fully merited by him.

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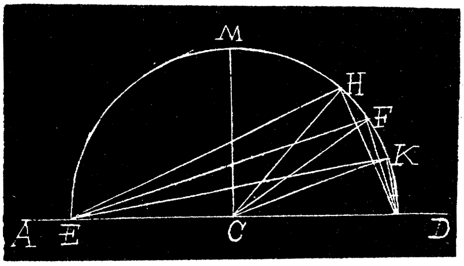
IS THE SUPPLEMENTARY ANGLE FINITE OR NOT?

By Professor JOHN N. LYLE, Ph. D., Westminster College, Fulton, Missouri.

If any individual angle whatever is greater than one right angle and less than two right angles, is it *finite* and is its supplementary angle, also, *finite*?



Let EMO be a semicircle and let the lines CH, CF, CK be drawn making the angles ECH, ECF, ECK greater than one right angle and less than two right angles.

Are these angles finite? The answer to this question depends upon the definition of a finite angle.

What, then, is a finite angle? We answer that it is one measured by an arc subtended by a chord that is a *finite* straight line. A *finite* straight line is one that has a beginning and a termination; that is, two ends. Geometrical definitions are *convertible*. Hence, a straight line that has a beginning and termination is *finite*.

Let us examine the angles in the light of these definitions. Each of the chords EH, EF, and EK that subtend the measuring arcs EMH, EMF, and EMK has two ends and by definition is finite. Each of the angles ECH, ECF, and ECK has the distinctive marks of a finite angle and, hence, by definition is *finite*.

Since these angles are less than two right angles and greater than one, the points H, F and K lie on the arc between M and O.

The supplementary angles HCO, FCO, and KCO are measured by the arcs HO, FO and KO whose chords are straight lines having two ends. Hence, by definition both chords and angles are finite. Some say that the angle KCO may become so small that it is *no longer finite* but *an infinitesimal!* But what is an infinitesimal? No definition of the term has appeared in any article published thus far in the AMERICAN MATHEMATICAL MONTHLY. Those who persist in using the term should explain with clearness and precision its meaning.

If the supplementary angle KCO is an infinitesimal (whatever that may mean), does the chord of the measuring arc KO retain both of its ends? If so, is not the chord still finite? If the chords is finite, is not the measuring arc, also, finite?

If the measuring arc KO is finite, has not the angle KCO measured by that arc distinctive finite marks? If KCO has distinctive finite marks is it not violative of the logical law of Non-Contradiction to deny this fact by calling the angle *an infinitesimal*? Furthermore, if the so-called infinitesimal has distinctive finite attributes wherein does it differ from a *finite* quantity?

Bledsoe, in his Philosophy of Mathematics, page 137, quotes Carnot as follows: "Leibnitz who was the first to give rules for the infinitesimal calculus, established it upon the principle that we can take at pleasure, the one for the other, two finite Magnitudes which differ from each other only by a quantity infinitely small.

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Still it was not free from objections; they reproached Leibnitz (1) with employing the expression infinitely small quantities without having previously defined it; (2) with leaving in doubt, in some sort, whether he regarded his calculus as absolutely rigorous, or as a simple method of approximation."

On page 168 of his Philosophy Bledsoe makes the following quotation from the Marquis de L'Hopital.—"We demand that we can take indifferently the one of two quantities for the other which differ from each other only by a quantity infinitely small; or (what is the same thing) that a quantity which is augmented or diminished by another quantity infinitely small can be considered as remaining the same."

If the difference between the angle sum of Lobatschewsky's triangle and two right angles is an infinitesimal, then according to De L'Hopital's "Demand" Lobatschewsky's angle sum is not less than two right angles but equal thereto.

In Euclid the angle sum of the triangle *ECH* is two right angles precisely. Hence, the angle sum has no supplementary angle. The assumption that there is a supplementary angle when in truth there is none proves that the words of the hypothesis do not correspond with geometrical fact. Is analytical symbolism properly used when employed to obliterate rather than to represent geometrical data? The hypothesis of *a zero angle* is a relic of logical barbarism whose presence in recent Mathematical literature indicates that the unfittest sometimes survives.

The triangle *ECH* just constructed is supposed to be in a plane—a surface absolutely without curvature.

If Lobatschewsky's triangle is not in a plane but in a pseudo-spherical surface, it can not be the triangle *ECH* which is in a plane.

If *ECH* is a Euclidian triangle, its angle sum is equal to two right angles and it can not at the same time be a Lobatschewsky triangle whose angle sum is less than two right angles.

The assumption that it can is in direct contravention of the logical laws of Non-contradiction and excluded Middle.

Lobatschewsky proved that the sum of the three angles of a rectilineal triangle can not be greater than two right angles. Riemann shows scant respect for Lobatschewsky's demonstration when he maintains that the angle sum of a rectilineal triangle is greater than two right angles.

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